

How noise in temperature translates to growth

Žarko Kovač^{1*}, Bror Jonsson³, Shubha Sathyendranath²,
Marin Vojković¹, Robert Brewin⁴, Qi Zheng⁴, Francois Ribalet⁵

¹ Faculty of Science, University of Split, Ruđera Boškovića 33, 21 000 Split, Croatia *zkovac@pmfst.hr

² Plymouth Marine Laboratory, Prospect Place, The Hoe, Plymouth PL1 3DH, United Kingdom

³ Ocean Processes Lab, University of New Hampshire, 7 College Road, Durham, 100190, New Hampshire, USA

⁴ Centre for Geography and Environmental Science, Department of Earth and Environmental Sciences, University of Exeter, Penryn Campus, Penryn, Cornwall, TR10 9EZ, UK

⁵ School of Oceanography, University of Washington, Seattle, WA, USA

The Simons Collaboration on Computational Biogeochemical Modeling of Marine Ecosystems
Annual Meeting, New York, 15th - 17th June 2026

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Noise in sea surface temperature

A typical sea surface temperature signal consists of a trend, seasonal cycle and noise. To get at the noise term the deterministic signal has to first be removed. We accomplish this with the following expression:

$$T(t) = \langle T \rangle + kt + A \sin(2\pi t/P + \varphi), \quad (1)$$

where $\langle T \rangle$ is the mean, k is the trend, A is the amplitude of the seasonal cycle, P is the annual period and φ is the phase. Below is a map of the standard deviation of noise in SST after the deterministic signal had been removed.

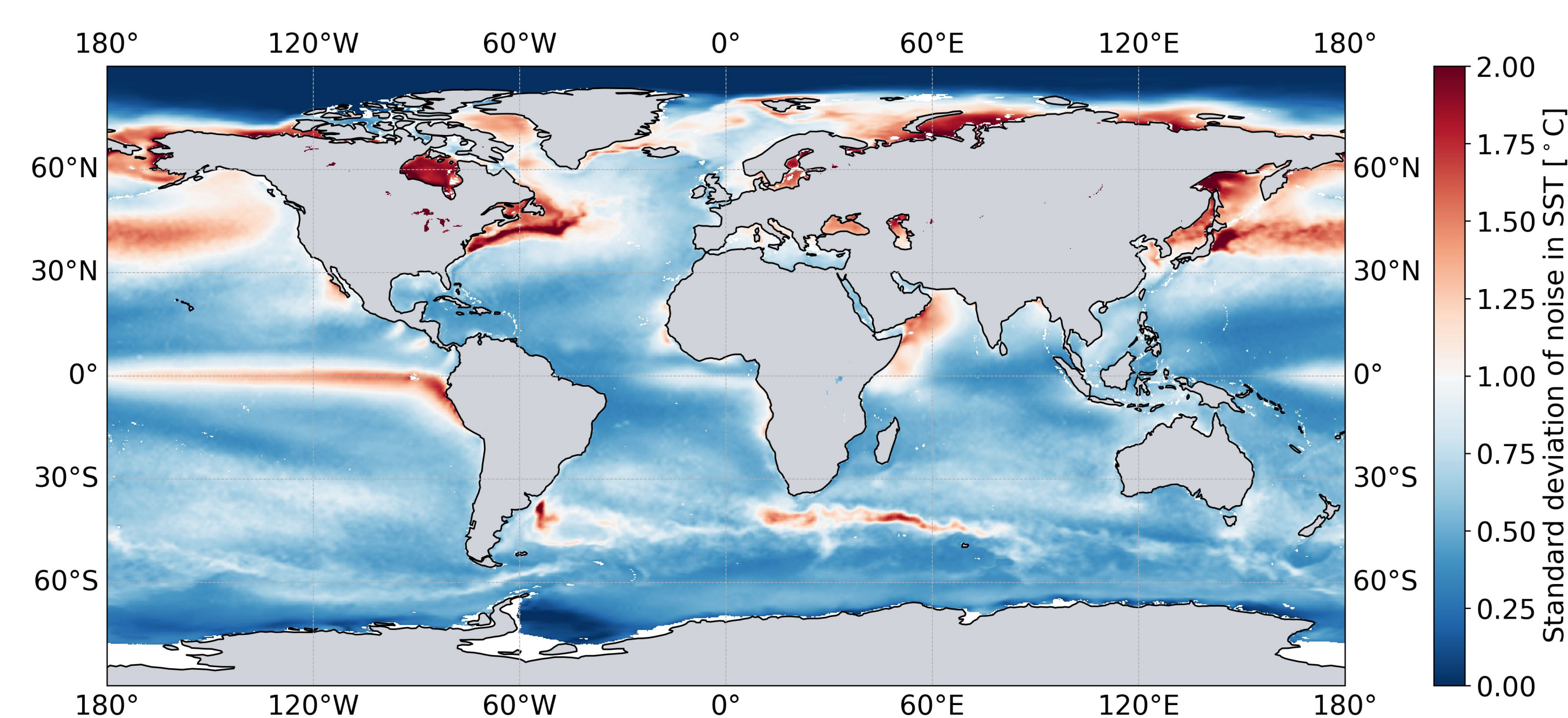


Figure 1: A map of standard deviation in sea surface temperature (SST) noise from ERA5 reanalysis data from the Copernicus Climate Data Store. At each point the mean, the trend and the seasonal cycle are removed by fitting (1) to the time series. Subsequently, a Gaussian distribution is fitted onto the remaining anomalies signal. The standard deviation of this fit is presented as a global map and is a measure of noise in the SST signal at each location.

Thermal performance function and noise

A recently published universal thermal performance function is of the following mathematical form (Arnoldi et al., 2025):

$$\mu(T) = \mu_m \exp\left(\frac{T - T_o}{T_c - T_o}\right) \left(1 - \frac{T - T_o}{T_c - T_o}\right), \quad (2)$$

where T_o is the optimal performance temperature, T_c is the critical temperature at which performance drops to zero and μ_m is the maximal growth rate at optimal temperature (Figure 2). Now, assume that temperature is a normally distributed random variable with mean $\langle T \rangle$ and standard deviation σ . With such a temperature distribution the average growth rate is given as:

$$\langle \mu(T) \rangle = \mu_m \exp\left(\frac{T - T_o}{T_c - T_o} + \frac{\sigma^2}{2(T_c - T_o)^2}\right) \left[1 - \frac{T - T_o}{T_c - T_o} - \frac{\sigma^2}{(T_c - T_o)^2}\right]. \quad (3)$$

By comparing the averaged growth function (3) to the original one (2) interesting conclusions can be drawn. After setting the condition that the averaged growth rate equals the instantaneous growth rate:

$$\mu(T) = \langle \mu(T) \rangle, \quad (4)$$

we arrive at the fragility temperature threshold:

$$T_f = T_o + (T_c - T_o) \left[1 - \frac{(\sigma^2/(T_c - T_o)^2) \exp(\sigma^2/2(T_c - T_o)^2)}{\exp(\sigma^2/2(T_c - T_o)^2) - 1}\right]. \quad (5)$$

This is the maximum temperature for which growth is antifragile. For temperature higher than T_f the effect of noise becomes detrimental for production and production becomes fragile. Subsequently, due to fragility, the temperature at which growth goes to zero becomes smaller by ΔT , given by:

$$\Delta T = \frac{\sigma^2}{T_c - T_o}. \quad (6)$$

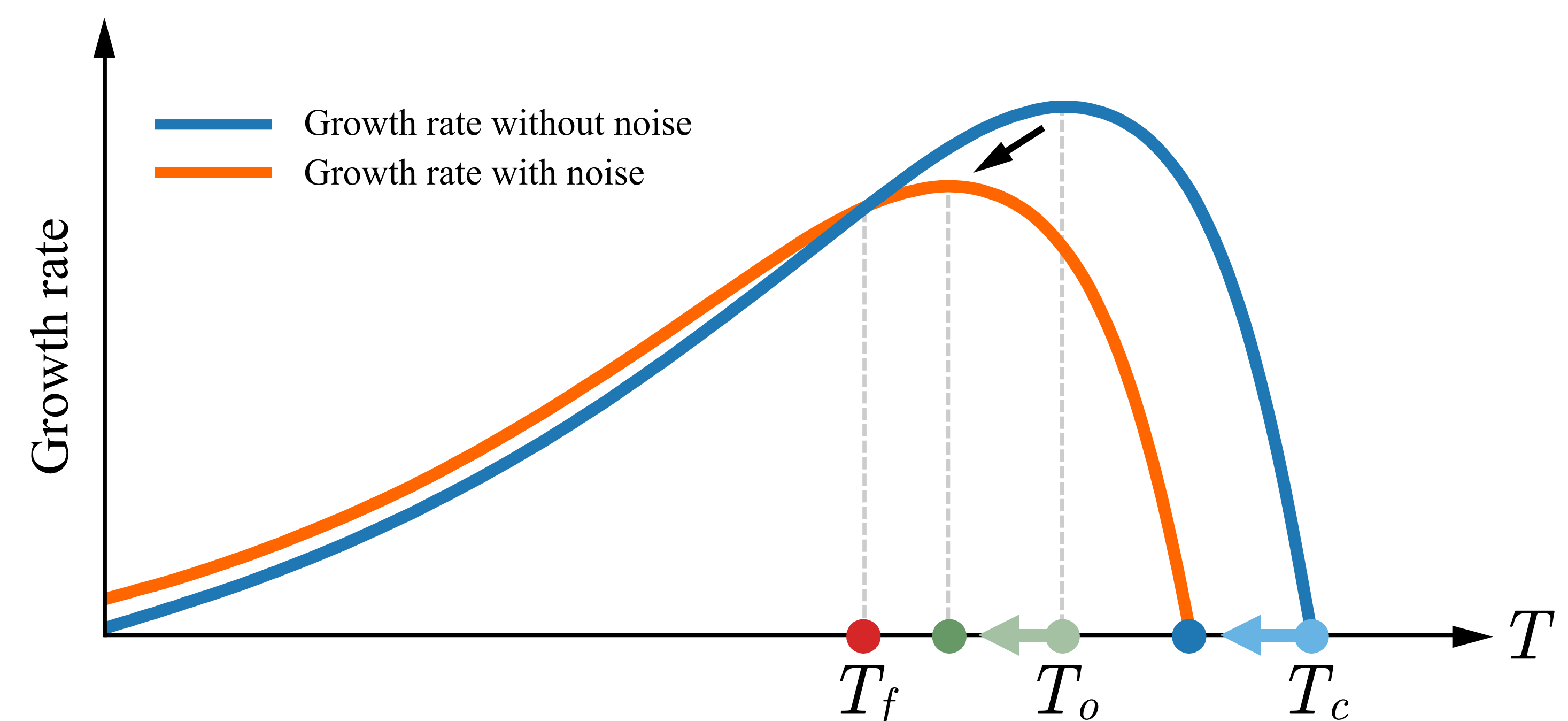


Figure 2: Plot of the universal thermal performance function (2) given in blue and its average (3) given in orange. The average is calculated by assuming a normally distributed noise around a fixed temperature. The thermal performance function has a maximum at the optimal temperature T_o and goes to zero at the critical temperature T_c . Upon averaging the optimal temperature and the critical temperature both shift to lower temperatures. The change ΔT in the critical temperature can be expressed analytically (6).

A simple stochastic growth model

The relevance of noise in the thermal performance function becomes evident when used in a growth model for mixed layer phytoplankton biomass B_t :

$$\frac{dB_t}{dt} = [\mu(T_t) - m] B_t, \quad (7)$$

where in this context $\mu(T_t)$ is a temperature dependent growth rate, with T_t being the mixed layer temperature and m being the loss rate. Let the temporal evolution of mixed layer temperature be given by the Ornstein–Uhlenbeck process:

$$dT_t = \lambda [T_e - T_t] dt + \sigma dW_t, \quad (8)$$

where T_e is the equilibrium temperature, λ is the mean reversion rate, σ is the noise intensity and W_t is Brownian motion (Wiener process). We have done simulations with the following stochastic model for temperature and coupled it to the growth equation. Sample results are shown below.

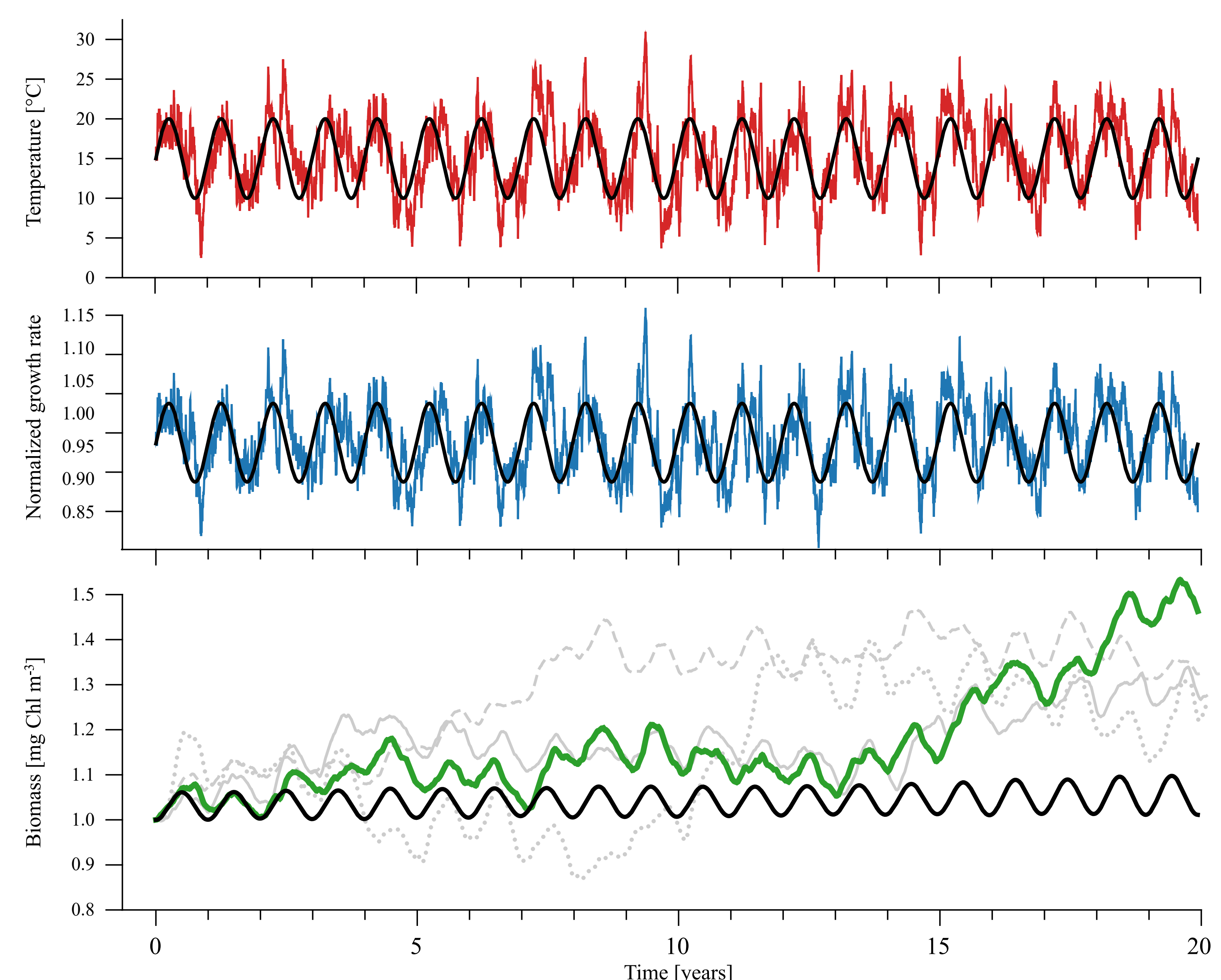


Figure 3: Example simulations of phytoplankton biomass with a simple growth model in which the growth rate is forced with a seasonally varying temperature (black curve in top panel) and seasonally varying temperature with noise (red curve). The growth rate without noise (black curve in middle panel) is bounded and leads to a stable seasonal cycle in biomass (black curve in bottom panel). When the temperature with noise is used to calculate the growth rate (blue curve in second panel) the seasonal cycle gets modified and can even exhibit long term trends due to antifragility (green curve). Grey curves in the bottom panel are sample paths from several simulations.

References

Arnoldi, J.-F., Jackson, A. L., Peralta-Maraver, I., Payne, N. L. (2025). A universal thermal performance curve arises in biology and ecology. *Proceedings of the National Academy of Sciences*, 122(43), 1-7. doi: 10.1073/pnas.2513099122