SIMONS COLLABORATION ON COMPUTATIONAL BIOGEOCHEMICAL MODELING OF MARINE ECOSYSTEMS
ANNUAL MEETING, ONLINE, 15TH - 17TH JUNE 2026

SECOND ORDER EFFECTS OF NOISE ON PRIMARY PRODUCTION

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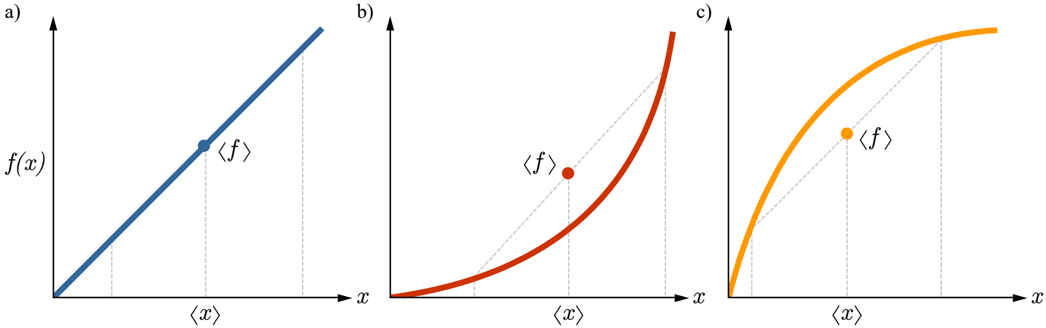
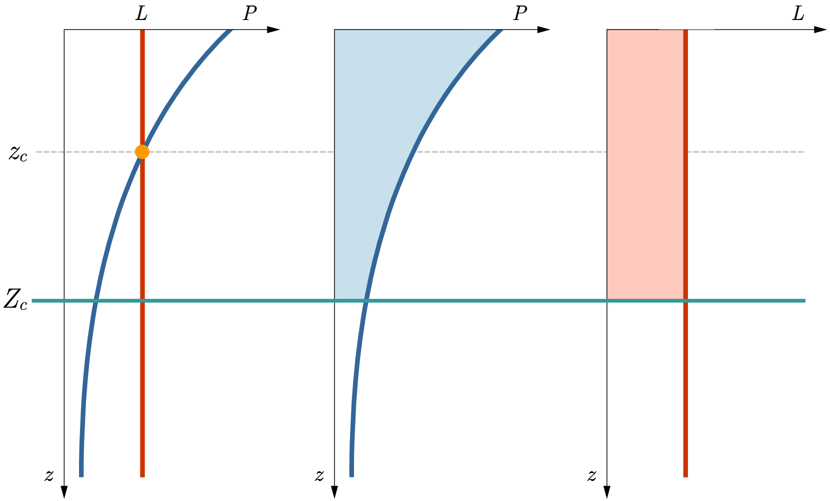
What is a second order effect?

Jensen's inequality (Jensen, 1906)



The compensation depth and the critical depth as defined by Sverdrup (1953)



Analytical expressions (Kovač et al., 2021)

Both the compensation depth and the critical depth have exact solutions:

$$z_c = \frac{1}{K} \ln A$$

$$Z_c = \frac{1}{K} \Big(W_0 \Big(- A e^{-A} \Big) + A \Big)$$

where the ratio of surface production to losses is given as:

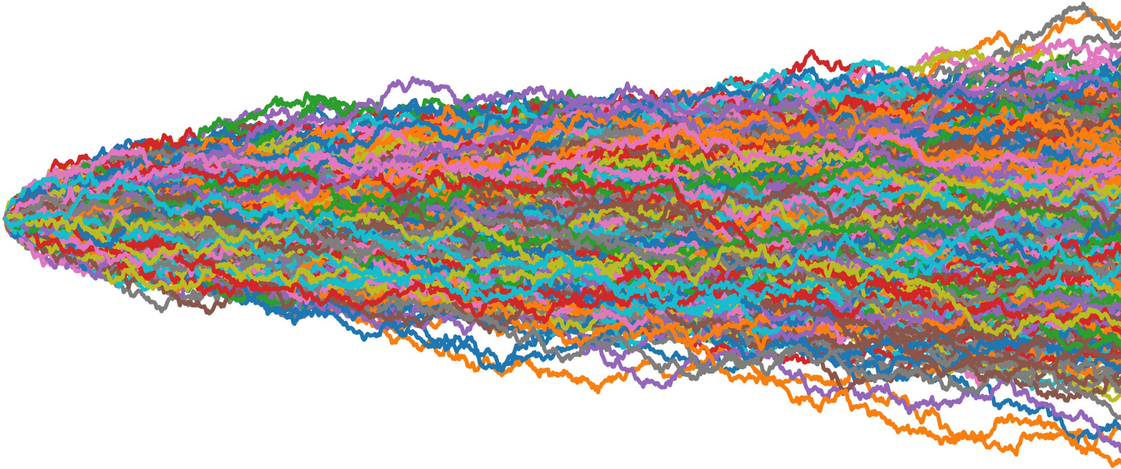
$$A = \frac{\alpha^B I_0}{L^B}$$

Switching onto the Lagrangian perspective:

$$\mathrm{d}Z_t = \sigma\,\mathrm{d}W_t$$

W_t is Brownian motion (in the mathematical sense).

Simulations for individual cell depths



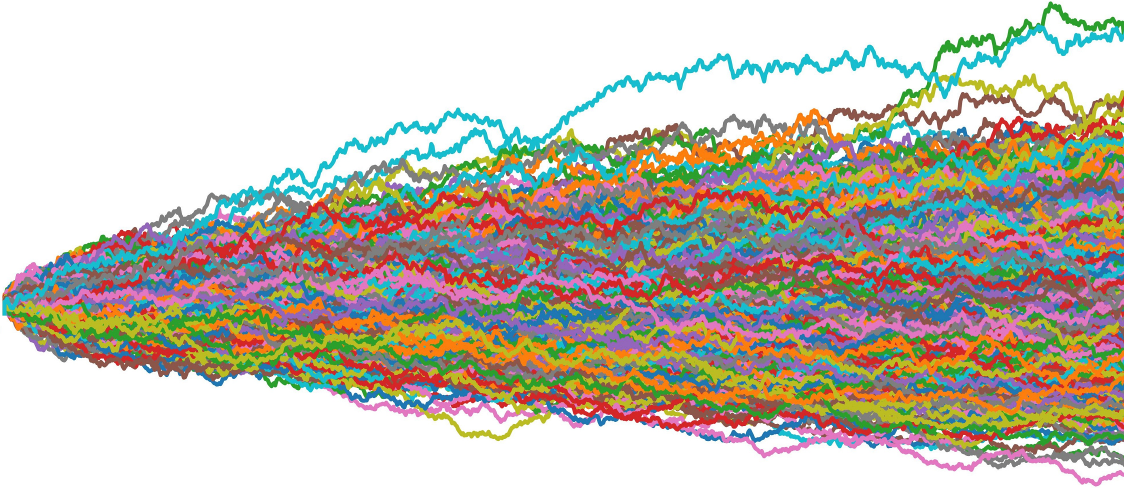
We wish to derive an equation for the production of the cell P_t^B, knowing:

$$P^B(z) = \alpha^B I_0 e^{-Kz}$$

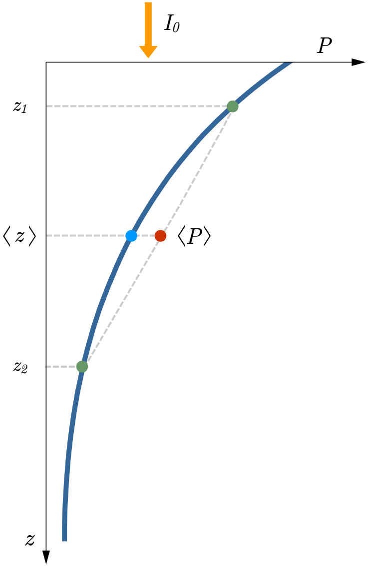
with the depth of the cell given by the solution to the prior equation:

$$Z_t = Z_0 + \sigma W_t$$

Simulations for irradiance experienced by individual cells



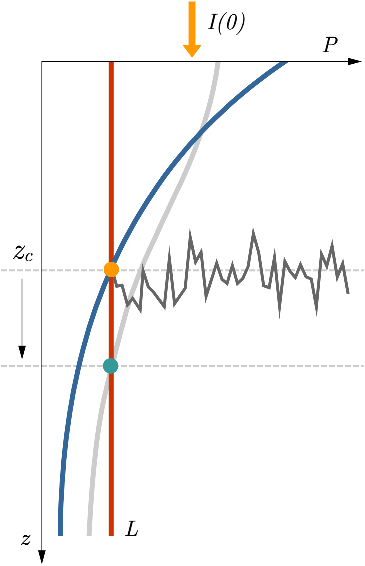
Ito's lemma and Jensen's inequality



$$\mathrm{d}I_t = \frac{\sigma^2 K^2}{2} I_t \,\mathrm{d}t - \sigma K I_t \,\mathrm{d}W_t$$

$$\langle I_t \rangle = I_0 e^{-K Z_0} \exp\left(\frac{\sigma^2 K^2}{2} t\right)$$

New solution for the compensation depth



$$\alpha^B I_0 e^{-K z_c} \exp\left(\frac{\sigma^2 K^2}{2} t\right) = L^B$$

$$z_c = \frac{1}{K} \ln\left(\frac{\alpha^B I_0}{L^B}\right) + \frac{\sigma^2 K}{2} D$$

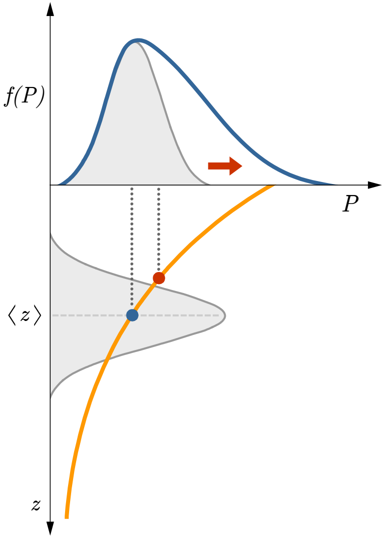
We can express the compensation depth as:

$$\zeta_c = \ln A + \zeta_\sigma$$

where a new nondimensional number ζ_σ emerges as:

$$\zeta_\sigma = \frac{\sigma^2 K^2}{2} D$$

Antifragility recognized in the Ito drift term

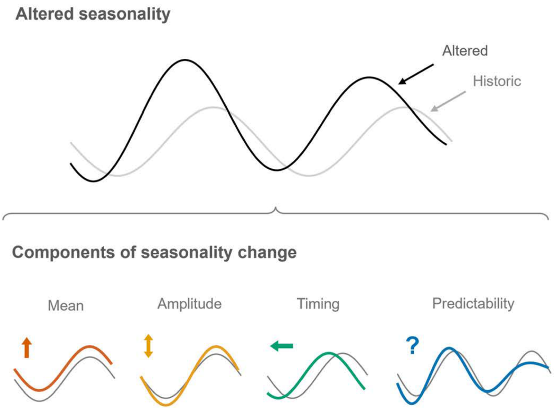


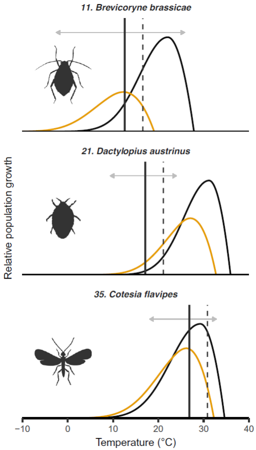
$$\mathrm{d}P_t^B = \frac{\sigma^2 K^2}{2} P_t^B \, \mathrm{d}t - \sigma K P_t^B \, \mathrm{d}W_t$$

$$f_P(P^B, t) = \frac{1}{P^B \, \sigma K \sqrt{2\pi t}} \exp\left[-\frac{\left(\ln\left(P^B / \alpha^B I_0 e^{-Kz}\right)\right)^2}{2\sigma^2 K^2 t} \right]$$

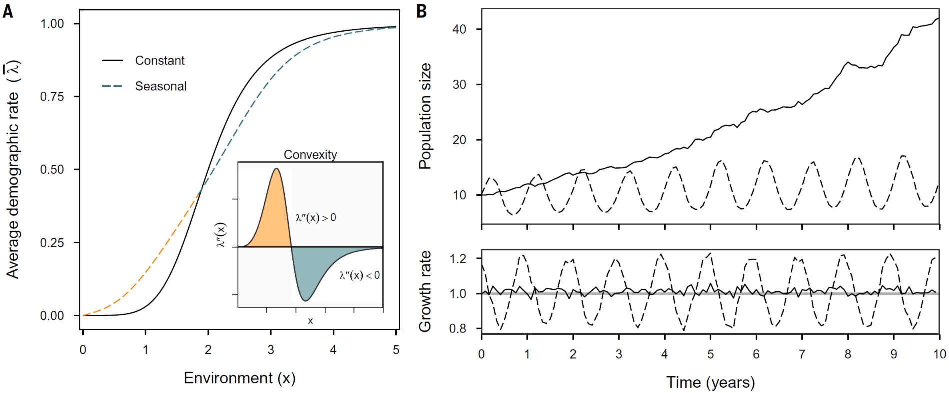
The same mathematical machinery can be used to study noise in temperature.

Changing seasonality on land (Hernández-Carrasco et al., 2025)





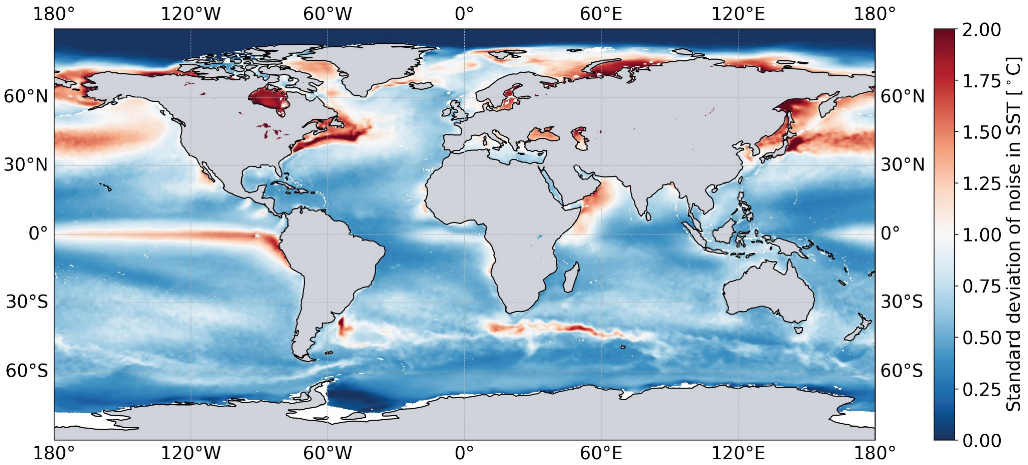
Effect of fluctuating growth rates on the long run population size



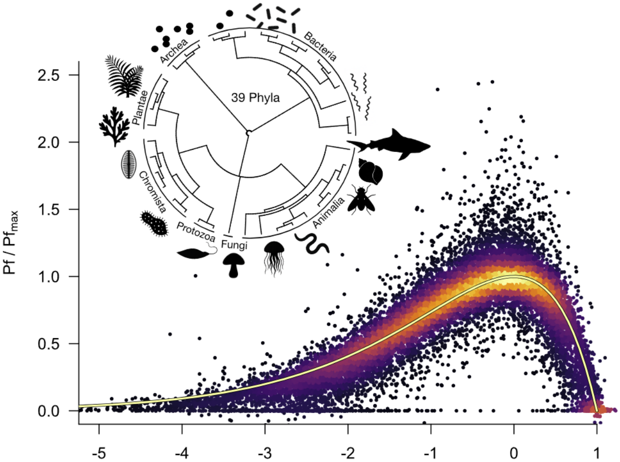
Hernández-Carrasco et al. (2025)

What about the ocean?

Noise in ERA5 monthly SST (preliminary results)

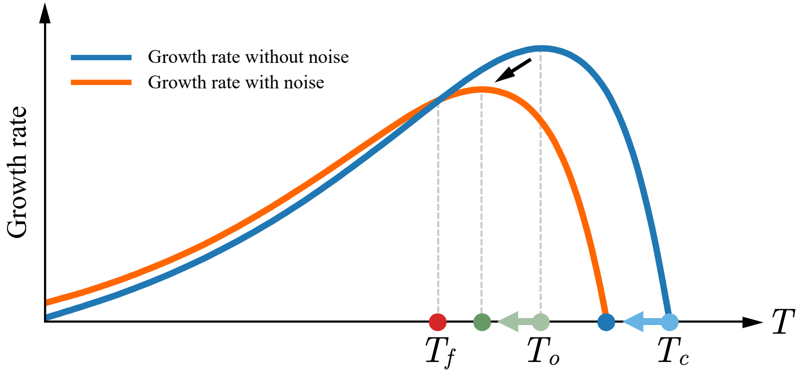


Universal thermal performance curve (Arnodli et al., 2025)



$$\mu(T) = \mu_m \exp\left(\frac{T - T_o}{T_c - T_o}\right)\left(1 - \frac{T - T_o}{T_c - T_o}\right)$$

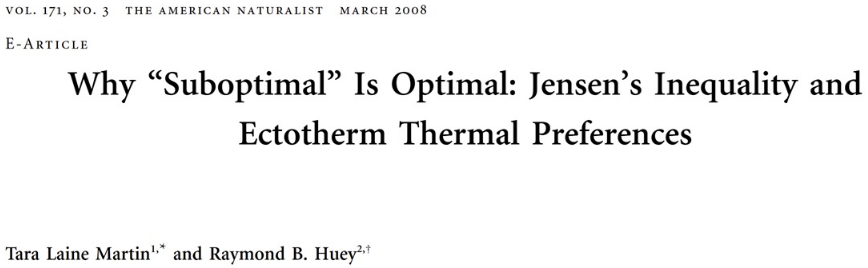
Jensen's inequality illustrated on the universal thermal performace curve



$$\langle \mu(T) \rangle = \mu_m \exp\left(\frac{T - T_o}{T_c - T_o} + \frac{\sigma^2}{2(T_c - T_o)^2}\right)\left[1 - \frac{T - T_o}{T_c - T_o} - \frac{\sigma^2}{(T_c - T_o)^2}\right]$$

The idea comes from Denny (2017).

A must read!



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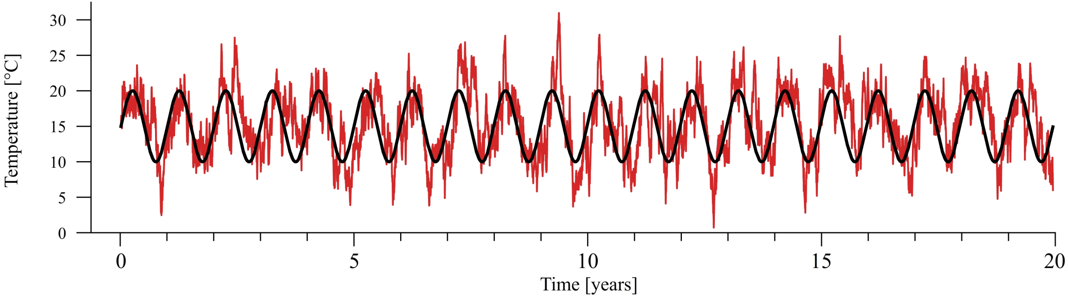
E-ARTICLE

Why “Suboptimal” Is Optimal: Jensen’s Inequality and Ectotherm Thermal Preferences

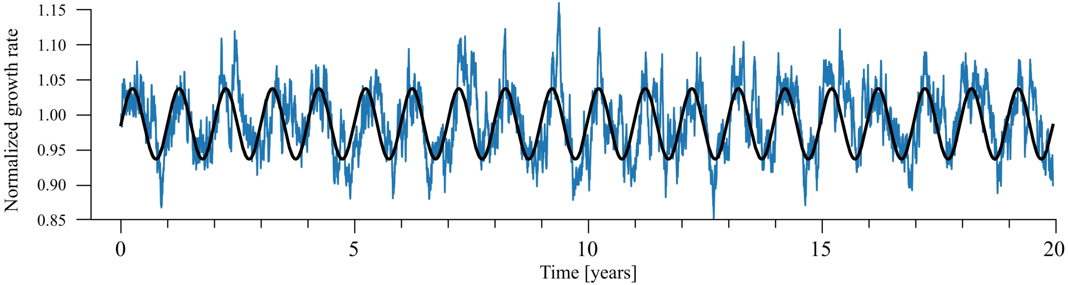
Tara Laine Martin[1,*] and Raymond B. Huey[2,†]

A simple growth model with noise in temperature



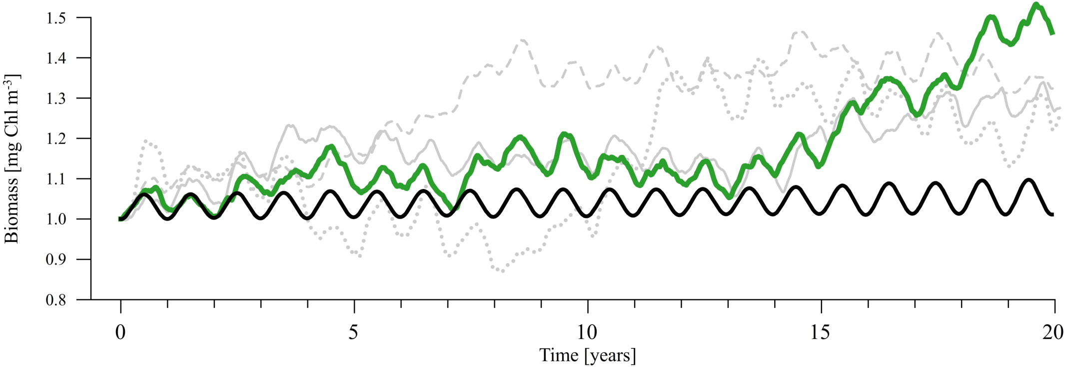
$$\mathrm{d}T_t = \lambda \Big[T_e - T_t \Big] \, \mathrm{d}t + \sigma \, \mathrm{d}W_t$$

A simple growth model with noise in temperature



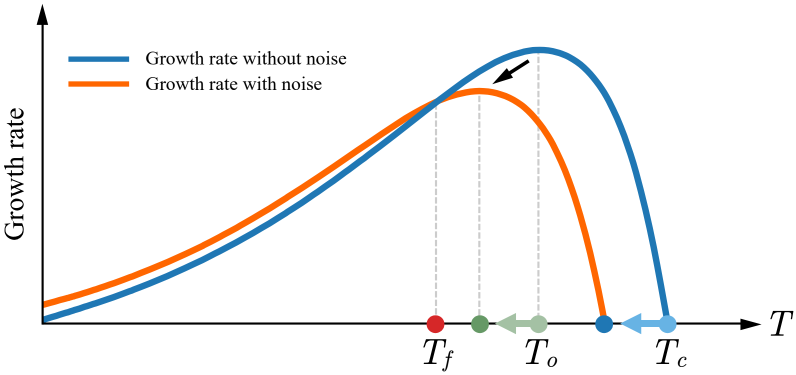
$$\mu(t) = \mu[T(t)]$$

A simple growth model with noise in temperature



$$\frac{\mathrm{d}B_t}{\mathrm{d}t} = \Big[\mu(T_t) - m\Big] B_t$$

Noise in temperature pushes the critical temperature down



$$\Delta T = \frac{\sigma^2}{T_c - T_o}$$

Key takeaway: From randomnes to determinism

Due to curvature in production functions, **randomnes** in the input **is transferred to a deterministic term** in the production response.

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Thank you!