

Bayesian Estimation of P–I Parameters with Uncertainty Propagation into Watercolumn Primary Production

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INTRODUCTION

We apply a hierarchical Bayesian framework to replicated ¹⁴C incubations from Bedford Basin (1975–76), a dataset previously analysed by classical least squares, offering a direct benchmark comparison. Unlike classical fitting, which yields a single best-fit parameter set per experiment, our approach returns full posterior distributions for all P–I parameters simultaneously (Sivia & Skilling, 2006), and demonstrates how these posteriors propagate directly into daily watercolumn production uncertainty.

P–I MODEL

We employ a photoinhibition model of Platt et al. (1980) with dark respiration:

$$P^B(I) = P_m^B \left(1 - e^{-\alpha^B I / P_m^B} \right) e^{-\beta^B I / P_m^B} - R^B.$$

The four parameters α^B , P_m^B , R^B and β^B set the low-light efficiency, light-saturated rate, dark respiration loss, and high-light decline respectively. Combining this curve with the full light field $I(z, t) = I_0 \sin(\frac{\pi t}{D}) e^{-Kz}$, where I_0 is surface noon irradiance and K the attenuation coefficient, and integrating over depth and the daylight period D gives the daily watercolumn production, $P_{Z,T}$. Each of the 8000 posterior draws produces one value of $P_{Z,T}$, yielding a full posterior distribution of daily watercolumn production per survey.

METHODS

Data. Replicated ¹⁴C incubations from Bedford Basin, Nova Scotia from 1975 to 1976 (Irwin & Platt, 1977); 107 surveys after dropping incomplete records.

Model. Full P–I model in PyMC. Per-survey α^B , P_m^B , R^B , β^B inferred jointly with group-level distributions. NUTS: zero divergences, $\hat{R} = 1.0$.

Watercolumn production. P–I curve evaluated on a Beer–Lambert depth grid ($K = 0.2 \text{ m}^{-1}$, typical of productive coastal inlets); I_0 approximated by the maximum incubation irradiance per survey, as no in-situ surface irradiance measurements were made in the original study and satellite ocean colour data were not available until 1978. Surface irradiance follows a sine curve over the daylight period; daylength computed astronomically (44.68°N).

RESULTS: PARAMETER POSTERIOR

Each parameter emerges as a full posterior distribution. α^B , P_m^B and R^B are sharp and fall in physiologically sensible ranges, consistent with the 1975/76 reference estimates. β^B concentrates near zero, indicating that high-light suppression is at most weak in these waters. The hierarchical structure acts across surveys: estimates for sparse or noisy surveys are pulled toward the group mean, reducing uncertainty without discarding information.

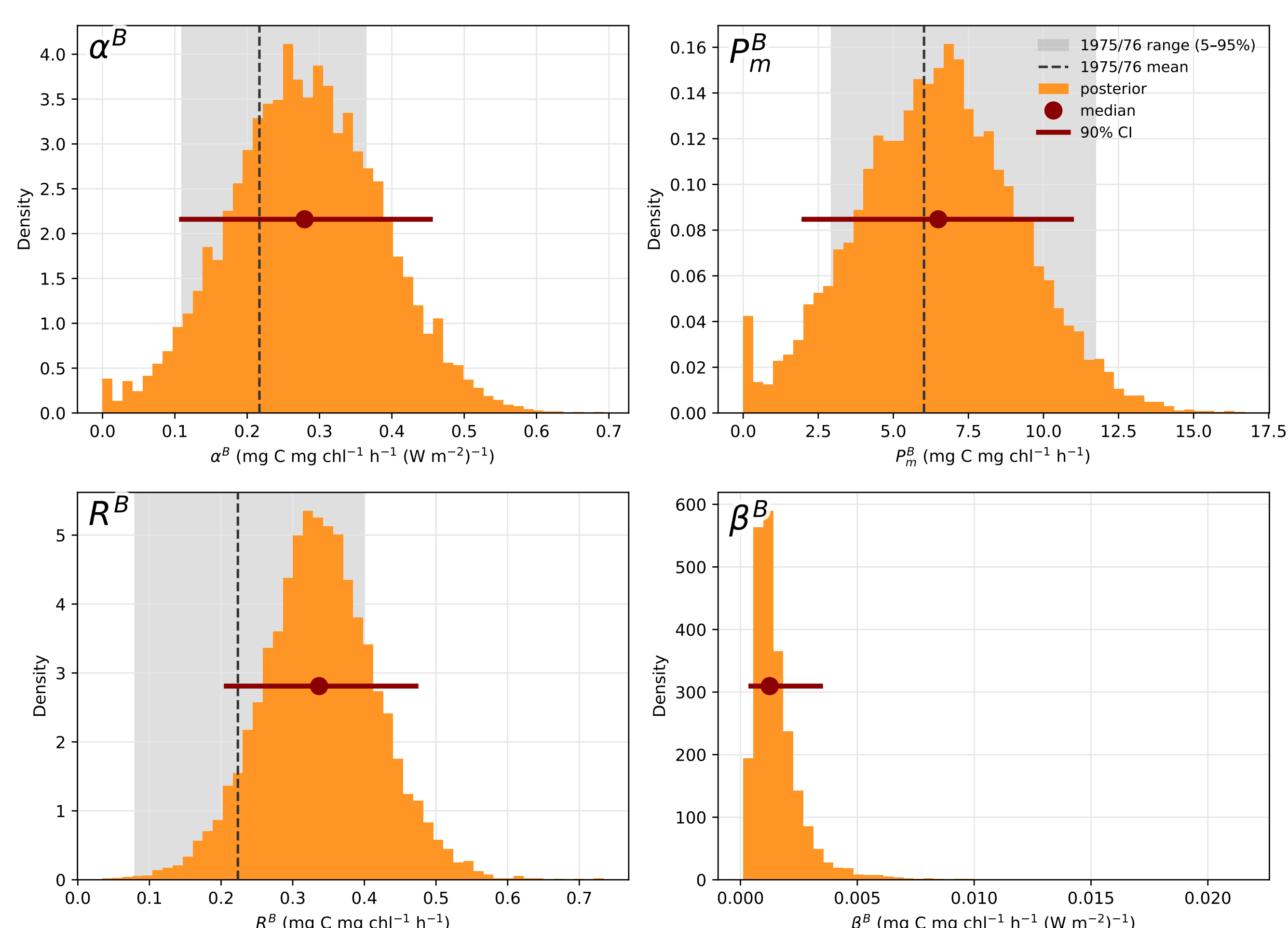


Figure 1: Group-level posterior distributions of the four P–I parameters α^B , P_m^B , R^B and β^B . Bars give the posterior density; markers show the posterior median with its 90 % credible interval. Dashed lines and shaded bands mark the mean and 5–95 % range of the 1975/76 reference estimates across surveys, where reported.

RESULTS: DAILY PRODUCTION PROFILES

Propagating the full posterior through the Beer–Lambert light field and integrating over the daylight period gives a daily normalized production profile $P_T^B(z)$ for every posterior draw. Summer profiles reach 100–175 mg C mg Chl^{−1} at the surface; the January profile is nearly flat near zero.

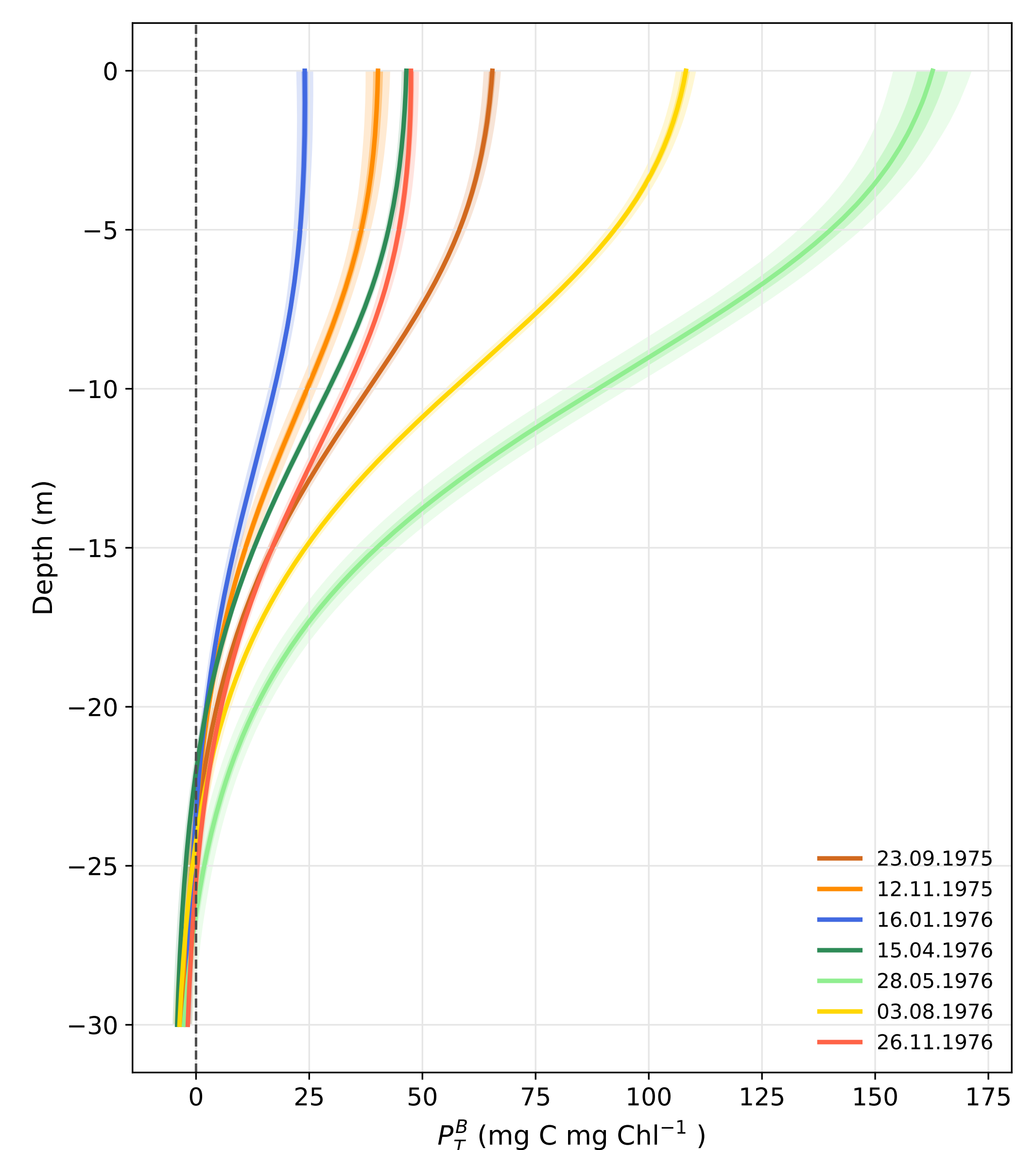


Figure 2: Daily normalized production for seven surveys. Colours: greens = spring, gold = summer, oranges/red = autumn, blue = winter. Bands: 25–75 % (darker) and 5–95 % (lighter) CI. Dashed: zero production.

RESULTS: WATERCOLUMN PRODUCTION

Depth-integrating over 0–30 m gives one estimate per survey. Production peaks in late spring and summer, collapses in winter, and recovers in spring. The 90 % credible envelope quantifies how uncertain the integrated estimate is at each point in the seasonal cycle.

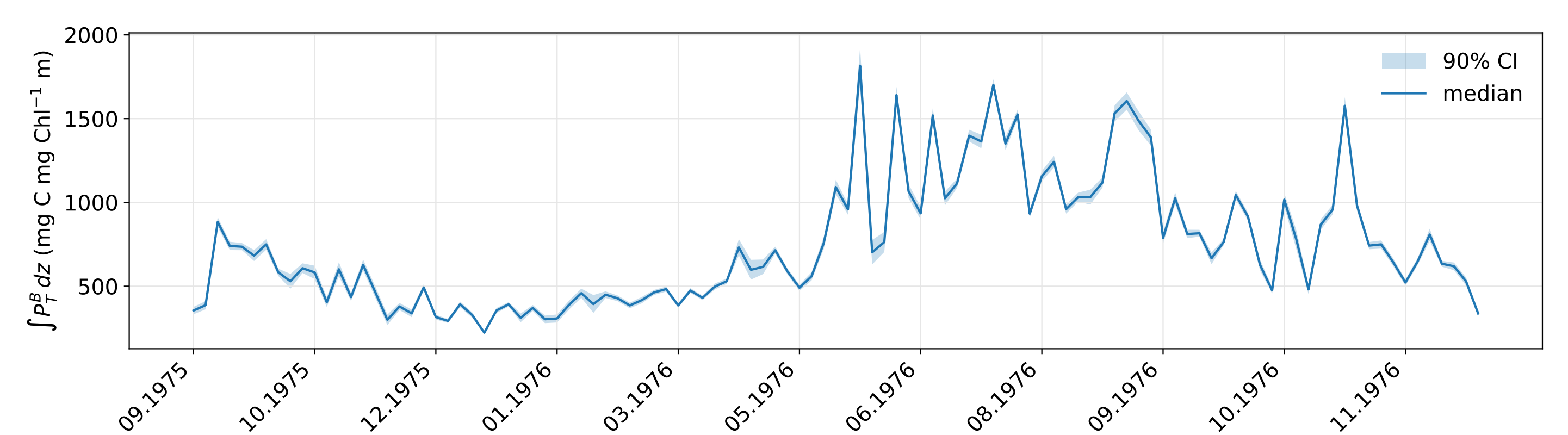


Figure 3: Daily watercolumn production $\int P_T^B dz$ (mg C mg Chl^{−1} m) for all 107 surveys spanning September 1975 to December 1976. The posterior uncertainty in α^B , P_m^B , R^B and β^B is propagated through the full depth and time integration. Line: posterior median. Band: 90 % credible interval.

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